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Stress Tensor Fluctuations and Stochastic Spacetimes

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The physics of the quantum stress tensor operator is discussed. Although the problem of defining the expectation values of this operator is reasonably well understood, the fluctuations around the mean value are not so well understood. It is shown that the stress tensor correlation function can be decomposed into three parts, one of which is finite and state dependent, one which is infinite in the coincidence limit but state independent, and a cross term which is both state dependent and infinite in the coincidence limit. Possible physical interpretations of each part are discussed. The fluctuations of the stress tensor in turn induce fluctuations of the spacetime geometry. The terms in the correlation function which are singular in the coincidence limit seem to produce drastic fluctuations of the geometry, leading to a stochastic spacetime. Whether these fluctuations are observable is an unanswered question.

1. INTRODUCTION: QUANTUM GRAVITY AND SPACETIME METRIC FLUCTUATIONS

The spacetime metric is the basic variable of gravity theory, analogous to the vector potential of electromagnetic theory. It is reasonable to expect that in any theory in which gravity obeys the laws of quantum mechanics, the metric will undergo quantum fluctuations. Metric fluctuation phenomena could be regarded as the essence of the quantum behavior of gravity. There are several reasons as to why these phenomena are likely to yield new physics.

One reason is that fluctuations of the metric can lead to fluctuations of the lightcone. Classical relativity theory forms a very tight logical structure with the concept of the lightcone at its heart. The classical lightcone determines which events may causally influence one another, and which events

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may not. Thus there is a crucial discontinuity across the lightcone. In classical relativity, massive particles may move at any speed less than that of light, massless particles must move exactly at the speed of light, and no physical particles may move faster. Thus even the slightest blurring of the distinction between timelike, lightlike, and spacelike intervals entails an enormous conceptual shift.

In general relativity, a special case of a lightcone is an event horizon, such as the boundary of a black hole. The horizon divides the universe into two parts: the region which can communicate with the black hole's exterior, and the region which cannot. Thus fluctuations of the horizon will fundamentally alter the concept of a black hole [1]. There is at least the theoretical possibility that information could leak out of the interior of a black hole. Horizon fluctuations could also alter the thermodynamic properties of black holes. Hawking [2, 3] showed in 1974 that a black hole emits a thermal spectrum of particles. This forged an elegant link between gravity and thermodynamics, which had been conjectured earlier by Bekenstein [4]. However, Hawking's derivation of this effect depends in a crucial way upon the precise character of a classical horizon. Even very slight fluctuations in the location of the horizon could invalidate this derivation and hence the link with thermodynamics [5].

There is another area of classical relativity which will need to be reassessed in the light of quantum metric fluctuations. These are the "singularity theorems" pioneered by Penrose and Hawking [e.g., 6]. These powerful theorems demonstrate the necessity of singularity formation in gravitational collapse under conditions that are quite reasonable in a classical system. These conditions include a restriction on the stress tensor of matter, such as local positivity of the energy density. This condition is obeyed by classical matter fields, although it may be violated over a limited region by quantum fields, as discussed above. For the collapse of a large object, such as a massive star, quantum violations of the energy conditions are unlikely to play a role until the collapse has proceeded to a point very close to a classical singularity. However, the proofs of the singularity theorems rely in a crucial way upon the notion of focusing of light rays. Metric fluctuations will tend to blur this focusing property. At the very least, the derivations of these theorems will have to be reexamined in any theory with a fluctuating metric.

Another intriguing possibility was raised by Pauli. He conjectured many years ago [7] that lightcone fluctuations could act as a universal regulator to remove the ultraviolet divergences of quantum field theory. His reasoning was based upon the observation that these divergences are due to the singularities of propagators on the lightcone. Hence if the lightcone were to be smeared out, perhaps the divergences would go away. This conjecture remains unproven, but is still a fascinating possibility [8–11].

There are two sources of quantum fluctuations of the spacetime metric: the intrinsic fluctuations of the quantized metric ("active fluctuations"), and fluctuations induced by fluctuations of a quantum matter field ("passive fluctuations"). In a generic situation, one expects both types of fluctuations to be present. A full theory of active fluctuations is tantamount to a full quantum theory of gravity, which is still far from realization. It is possible to develop models of lightcone fluctuations due to linearized quantum gravity which lead to a reasonable physical picture [12, 13] of a fluctuating lightcone. However, this topic is beyond the scope of the present work. Here we will focus our attention upon the passive metric fluctuations, and hence upon stress tensor fluctuations.

2. THE QUANTUM STRESS TENSOR

The stress tensor $T_{\mu\nu}$ of classical physics carries the information about the energy density, pressures, and stresses due to a classical field such as the electromagnetic field. When we pass from a classical field to a quantized field, a number of new features arise. First, the formal expectation value of the stress tensor components in an arbitrary quantum state is divergent. This is a reflection of the infinite zero-point energy of a quantum field. Such a field is an infinite collection of quantum harmonic oscillators, each with a ground-state energy proportional to the oscillator's frequency.

For a system with a finite number of degrees of freedom, such as the atoms in a crystal lattice, this energy is finite and observable. For a quantum field, it cannot be taken literally. Otherwise, one would have all of space filled with an enormous energy density. In most areas of physics, one can evade the question with the response that only changes in energy are observable, so what counts is just the shift in the energy density when we change the quantum state. However, once this energy density becomes the source of a gravitational field, this answer is no longer adequate. The actual value of the energy density is the source of gravity, and is hence an observable quantity. Indeed, we know from cosmological data that the average energy density in the universe must be quite small. Anything greater than about 10^{-29} g/cm^3 would have a noticeable effect on the current rate of expansion of the universe [e.g., 14].

Thus we are forced to conclude that the quantum stress tensor must be redefined by a renormalization procedure. This topic has been thoroughly investigated in recent decades, and can be considered to be a solved problem [15]. In the case of a flat spacetime, this renormalization takes the simple form of subtracting the vacuum expectation value. Thus if $\langle T_{\mu\nu}\rangle$ denotes the expectation value of the stress tensor in an arbitrary quantum state, and $\langle T_{\mu\nu}\rangle_0$ that in the Minkowski vacuum state,

$$\langle T_{\mu\nu}\rangle_{\rm ren} = \langle :T_{\mu\nu}: \rangle = \langle T_{\mu\nu}\rangle - \langle T_{\mu\nu}\rangle_0 \tag{1}$$

is the renormalized expectation value, which is finite everywhere for physically well-behaved quantum states.

The famous Casimir effect [16] is an illustration of the nontrivial physics associated with a renormalized stress tensor. One considers a pair of parallel, perfectly reflecting plates. The quantum fluctuations of the electromagnetic field produce a force of attraction between these plates which is observable [17, 18]. Here one is seeing an effect of the shift in energy between a configuration with the plates and one without the plates. One can think of the plates as shifting the vacuum fluctuations by a finite amount. More precisely, what has shifted are the fluctuations of the electric and magnetic fields in the region between the plates. The electric field, for example, is fluctuating around a mean value of zero, so that $\langle \mathbf{E} \rangle = 0$, but $\langle \mathbf{E}^2 \rangle \neq 0$. Of particular interest is that the renormalized energy density between the plates is negative; $\langle \rho_{\rm ren} \rangle < 0$. (This is consistent with the attractive nature of the Casimir force.) In classical physics, a negative energy density for the electromagnetic field is not possible; the energy density is proportional to $\mathbf{E}^2 + \mathbf{B}^2$, a positive-definite quantity. The energy density of a quantum stress tensor evades this constraint by virtue of being a difference between two ill-defined quantities. Some simple examples of states with negative energy are generated in the moving mirror models of Fulling and Davies [19, 20].

The existence of negative energy density in quantum field theory raises a number of interesting and unsolved problems as to the nature of the gravitation field to which such energy will give rise. There are a number of bizarre solutions of Einstein's equations which become possible if there are no restrictions at all on the magnitude and extent of negative energy. These include "traversable wormholes" [21], which could act as shortcut tunnels to distant parts of the universe, and "warp drive" spacetimes [22] in which faster-than-light travel becomes possible. Both of these could conceivably be converted into a "time machine" [23]. Fortunately (or unfortunately) quantum field theory does place some strong constraints on the magnitude and extent of negative energy density [24, 25], which seem to severely limit these exotic possibilities.

3. STRESS TENSOR FLUCTUATIONS

We now come to the key problem of this paper: how to define the quantum fluctuations of the stress tensor for a matter field. In principle, the problem is no different from that of defining the fluctuations in any other variable. One wishes to compare the average of the square of the variable with the square of its average. In quantum field theory we encounter the special problem of ultraviolet divergences. We have already seen this above when the issue of defining the expectation value of the stress tensor or of the squared electric field was addressed. In that case, it is possible to circumvent the divergence problem with the simple subtraction of a state-independent quantity. The study of stress tensor fluctuations requires one to be able to define expectation values of products of stress tensor operators, such as $\langle T_{\mu\nu}(x)T_{\rho\sigma}(x')\rangle$, which is considerably more challenging. This problem is difficult enough in flat spacetime, so our attention will be restricted to that case. Let $T(x) = :\phi_1(x)\phi_2(x):$ be a stress tensor component (a normal-ordered quadratic operator), such as the energy density. Here ϕ_1 and ϕ_2 are free quantum fields or derivatives of free fields. The expectation value of T in any physically realizable state is finite. In Minkowski spacetime, normal ordering simply means subtraction of the expectation value in the Minkowski vacuum state:

$$T(x) = :\phi_1(x)\phi_2(x): = \phi_1(x)\phi_2(x) - \langle\phi_1(x)\phi_2(x)\rangle_0 \tag{2}$$

Now consider a product of stress tensors at different points. It may be shown using Wick's theorem that

$$T(x)T(x') = S_0 + S_1 + S_2 \tag{3}$$

where

$$S_0 = \langle\phi_1(x)\phi_1(x')\rangle_0\,\langle\phi_2(x)\phi_2(x')\rangle_0 + \langle\phi_1(x)\phi_2(x')\rangle_0\,\langle\phi_2(x)\phi_1(x')\rangle_0 \tag{4}$$

$$\begin{aligned} S_1 = {} & :\phi_1(x)\phi_1(x'):\,\langle\phi_2(x)\phi_2(x')\rangle_0 + :\phi_1(x)\phi_2(x'):\,\langle\phi_2(x)\phi_1(x')\rangle_0 \\ & + :\phi_2(x)\phi_1(x'):\,\langle\phi_1(x)\phi_2(x')\rangle_0 + :\phi_2(x)\phi_2(x'):\,\langle\phi_1(x)\phi_1(x')\rangle_0 \end{aligned} \tag{5}$$

and

$$S_2 = :\phi_1(x)\phi_2(x)\phi_1(x')\phi_2(x'): \tag{6}$$

Thus the operator product $T(x)T(x')$ consists of a purely vacuum part S_0, a fully normal-ordered part S_2, and a part S_1 which is a cross term between the vacuum and normal-ordered parts.

As an aside, let us here briefly discuss the operator ordering problem in quantum theory. It is well known that observables in quantum mechanics are associated with operators that generally do not commute with one another. This leads to an essential ambiguity in constructing a quantum theory from a classical theory. Various proposals have been made for resolving this ambiguity [26]. Here we will adopt the symmetrization approach. Thus if ϕ_1 and ϕ_2 are noncommuting operators, the classical expression $\phi_1\phi_2$ should be replaced by the quantum operator $\frac{1}{2}(\phi_1\phi_2 + \phi_2\phi_1)$. Note that the use of normal ordering (in which creation operators are moved to the left and annihilation operators are moved to the right) is a separate issue. For quadratic operators, as in Eq. (2), normal ordering is just a shorthand for subtraction of the Minkowski vacuum expectation value. For quartic operators, the decomposition of Eq. (3) is a mathematical identity. The challenge here will be to give a physical interpretation of each piece.

So long as x and x' are distinct non-lightlike-separated points, all three terms on the right-hand side of Eq. (3) have finite expectation values. However, in the coincidence limit, $x' = x$, only the fully normal ordered part remains finite. The purely vacuum part does not pose a serious problem, as we can restrict our attention to the difference in the expectation value in an arbitrary state and in the vacuum state:

$$\langle T(x)T(x')\rangle - \langle T(x)T(x')\rangle_0 = \langle S_1\rangle + \langle S_2\rangle \tag{7}$$

Although $\langle S_2\rangle$ is always finite, $\langle S_1\rangle$ is both infinite and state dependent in the coincidence limit. Thus $\langle T^2(x)\rangle$ is not well defined. This means that it is not possible to define such quantities as the squared energy density or pressure in a straightforward manner so long as this term is present.

There seem to be two possible approaches to this problem. One is to impose some additional renormalization to remove the infinity, and the other is to replace the local operator products by finite spatially or temporally averaged quantities. If one adopts the former approach, the simplest possibility is to drop the cross term S_1 and use only the fully normal ordered part. This approach was used in refs. 27–29, where it was shown that one obtains the correct classical limit in the sense that

$$\langle :T(x)T(x'):\rangle = \langle S_2\rangle = \langle T(x)\rangle\,\langle T(x')\rangle \tag{8}$$

if the quantum state is a coherent state. This would imply that a classical field excitation, which is described by a coherent state, exhibits no quantum fluctuations in its stress tensor. (The generalization of this approach to curved spacetimes has been discussed by Phillips and Hu [30].)

3.1. Physics of the Normal Ordered Term

Let us first consider the fluctuations of the energy density which arise from the fully normal ordered term S_2. Let

$$\rho = :T_{tt}: \tag{9}$$

be the (normal-order) energy density operator. It has a finite expectation value everywhere in all physically realizable quantum states. Let $\langle\rho^2\rangle_2 = \langle:\rho^2:\rangle$ be the fully normal ordered energy density at a point. Define the dimensionless measure of the energy density fluctuations:

$$\Delta = \frac{\langle \rho^2 \rangle_2 - \langle \rho \rangle^2}{\langle \rho^2 \rangle_2} \tag{10}$$

As it should be, this quantity is small in a system which is nearly classical, such as that described by a coherent state with large amplitude.

However, in other states such as a thermal state, there can be large energy density fluctuations. When these fluctuations couple to the gravitational field, large metric fluctuations can arise. Consider a box filled with a thermal bath of photons. The stress tensor fluctuations not only cause fluctuations in the local gravitational field of the box, but also radiate gravitons. The rate of energy loss by graviton emission was calculated in ref. 28, where it was shown that the emitted power from a sphere of radius R is

$$P = (1.1 \times 10^{-65}\ \text{ergs/s}) R^3\, T \ln(160 RT) \tag{11}$$

where T is the temperature in degrees Kelvin, and R is in centimeters. Under normal circumstances, this effect is unmeasurably small. Nonetheless, it can have a significant effect. Some authors [31, 32] have conjectured the existence of regions of the universe, such a confined exotic phase of matter, that couple very weakly with the rest of universe. These regions would still be expected to cool slowly by graviton emission, and if they were formed in the early universe, could have an internal temperature of no more than 10 MeV at the present time. If such regions exist, their cooling is a nontrivial consequence of quantum gravity effects.

It is also of interest to examine the energy density fluctuations in a single mode squeezed state $|\psi\rangle = |z, \zeta\rangle$. This two-parameter family of states [33] includes the classical-like coherent states in the limit that $\zeta = 0$, but also includes states with very nonclassical properties when $|\zeta|$ is large. The latter include states which exhibit local negative energy densities. If one calculates Δ for a squeezed state, the result is always at least of order unity whenever the energy density $\langle \rho \rangle$ is negative [29]. This suggests that negative energy densities are highly fluctuating. Furthermore, the gravitational field produced by a system with a negative energy density at some points will be a wildly fluctuating field. A similar result applies to the Casimir effect. It is shown in ref. 29 that the Casimir energy density is always undergoing significant fluctuations. For the case of a scalar field, one finds that $\Delta \geq \frac{1}{3}$ for arbitrary boundary conditions. Given that it is very difficult to study the Casimir effect for generic boundary conditions, this is a remarkable result and one which holds regardless of whether the Casimir energy density is positive or negative. It implies that the gravitational force on test particles produced by Casimir energy density is necessarily undergoing large fluctuations. The Hawking flux from a black hole may also be shown to be undergoing fluctuations of order unity on time scales of the order of the light travel time across the black hole [34]. This suggests a picture in which the mean mass of the black hole monotonically decreases, but the actual mass undergoes rapid fluctuations around this mean value. Thus the normal ordering approach seems to lead to a physically consistent picture.

3.2. Physics of the Cross Term

It is still natural to wonder whether or not there is some nontrivial physics hidden in the S_1 cross term. If so, it can only be manifest when one uses time-averaged quantities as the physical observables, rather than local quantities. Various approaches to this subject have been offered, including that of Barton [35], who uses time-averaged stress tensor components to study the fluctuations of the Casimir force. The stress tensor operator is both the source of gravity and the source of forces on material bodies. Thus one might hope to learn about the quantum nature of gravity by studying the more experimentally accessible force fluctuations on mirrors.

There is one difference that needs to be borne in mind, however. Real mirrors become transparent to electromagnetic radiation at about the plasma frequency of the material in the mirror, typically in the ultraviolet part of the spectrum. Thus there is a natural cutoff which prevents the quantum fluctuation effects from ever becoming too large. The remarkable property of the Casimir force is not that it is finite, but rather that it is independent of this cutoff. Whether the fluctuations of the force are also cutoff independent is not yet clear. In the case of gravity, there does not seem to be any natural cutoff short of the Planck scale. If one computes the contribution of the cross term to such physical effects as graviton emission by a thermal bath using a Planck scale cutoff, the result will be far too large. If such a cutoff actually controlled the scale of the metric fluctuations, quantum gravity effects would have already been observed. Microwave ovens, for example, would not be able to function because of the enormous energy dissipation from graviton emission. Thus if the cross term is to be taken seriously, we need an approach which leads to cutoff-independent results. Here we will outline such an approach based upon the Langevin equation.

To illustrate the basic ideas, let us consider a mirror with mass m which starts from rest at time $t = 0$. The mean velocity and mean squared velocity at $t = \tau$ are

$$\langle v \rangle = \frac{1}{m} \int_0^{\tau} \langle F \rangle \, dt \tag{12}$$

and

$$\langle v^2 \rangle = \frac{1}{m^2} \int_0^\tau \int_0^\tau \langle F(t)F(t') \rangle \, dt \, dt' \tag{13}$$

Here F is the force operator describing the force exerted by a quantum field upon the mirror. The force operator is in turn formed from components of the stress tensor operator. For the case of a perfectly reflecting mirror in one spatial dimension, it may be expressed as

$$F = 2 \, :T^{rt}: \tag{14}$$

where $:T^{rt}: = T^{rt}$ is the energy flux impinging upon the mirror. The force correlation function $\langle F(t)F(t') \rangle$ has a decomposition of the form of Eq. (3). The purely vacuum term S_0 can be ignored on the grounds that we are interested in the change in the mean squared velocity when we change the quantum state from the Minkowski vacuum to some other state. The fully normal ordered part S_2 poses no fundamental problems because it is finite everywhere. However, the $\langle S_1 \rangle$ cross term poses the problem of being singular at $t = t'$. Its contribution to $\langle v^2 \rangle$ is of the form

$$\langle v^2 \rangle_1 = \int_0^\tau \int_0^\tau \frac{F(t, t')}{(t - t')^2} \, dt \, dt' \tag{15}$$

where $F(t, t')$ is a finite function which depends upon the choice of quantum state. This integral is formally divergent. However, there is a trick which may be employed to redefine it to be a finite integral. This trick has been used by various authors under the rubrics of "generalized principle value" [36] or "differential regularization" [37]. In any case, it involves writing the singular factor as a derivative of a less singular function, and then integrating by parts. If we assume that F and its first two derivatives vanish as $t \to 0$ and $t \to \tau$, then the surface terms vanish, and we can write

$$\langle v^2 \rangle_1 = \frac{1}{2} \int_0^\tau \int_0^\tau [\partial_t \, \partial_{t'} \, F(t, t')] \ln[(t - t')^2] \, dt \, dt' \tag{16}$$

The singularity of the integrand at $t = t'$ is now clearly integrable.

The resulting integral may be evaluated explicitly for various special cases [34], such as a single mode coherent state or a thermal state. In both cases, one finds that $\langle v^2 \rangle$ grows linearly in time:

$$\langle v^2 \rangle \propto \tau, \qquad \tau \to \infty \tag{17}$$

In the case of a thermal state at temperature T in one spatial dimension,

$$\langle v^2 \rangle_1 \sim \frac{16\pi}{33} \frac{T^3}{m^2} \tau \tag{18}$$

whereas the contribution from the normal ordered term is of the same form, but 1/3 as large:

$$\langle v^2 \rangle_2 \sim \frac{16\pi}{99} \frac{T^3}{m^2} \tau \tag{19}$$

Thus the cross term yields the dominant contribution here and apparently cannot be ignored if the procedure we have adopted for defining it is correct.

The same basic principles used in the problem of a mirror in one spatial dimension may also be applied in more general contexts. For example, the cross term contributes to the long-term mass fluctuations of an evaporating black hole in a way similar to the normal ordered term, but the effect of the former is three times that of the latter. In three spatial dimensions, the analog of Eq. (15) contains a singular factor of $(t - t')^{-4}$, and the analog of Eq. (16) contains four derivatives of F, but otherwise one can carry through the same calculations.

In the case of a coherent state, the effect we are discussing is just that of radiation pressure fluctuations, which are of great interest to designers of laser interferometer detectors of gravity waves, such as LIGO. These fluctuations were first treated by Caves [33] using a different approach. It will be of interest to apply the method outlined here to specific situations such as that of LIGO and compare the results with those of Caves.

The stress tensor describes both the force on material bodies as well as the source of the gravitational field. If one retains the cross term, then the gravitational field must be undergoing violent fluctuations on small time scales. The physical manifestation of these fluctuations is unclear, but must somehow show up in the Brownian motion of test particles in this fluctuating field.

3.3. Physics of the Vacuum Term?

Finally, let us briefly address the question as to whether there can be any observable consequences of the pure vacuum term S_0 in Eq. (4). As noted earlier, this term is independent of the quantum state and will hence drop out if the quantity one is measuring involves a difference between two states, as is typically the case. The vacuum term describes the irreducible quantum fluctuations which are always present and normally unobservable. This term is also the most singular in Eq. (3), diverging as $(x - x')^{-8}$ when $x' \to x$.

One can remove this vacuum divergence if instead of operators at a spacetime point, one defines smeared operators by integration over a finite region of space and time. This procedure is employed in axiomatic approaches to field theory [38]. However, here the smearing is a formal procedure intended to produce operators with nice mathematical properties. If one wishes to give a physical motivation for the smearing, it is necessary to decide what sets its spatial and temporal scales. There does not seem to be an obvious answer to this question.

One might conjecture that the backreaction of the stress tensor fluctuations on the spacetime geometry itself defines a characteristic averaging scale. Suppose that we average the quantum stress tensor operators over a region of linear dimension ℓ, resulting in local root-mean-squared energy densities of the order of ℓ^{-4}. These fluctuations in turn induce passive metric fluctuations with a typical size $s \approx \ell^2/\ell_{\rm Pl}$, where $\ell_{\rm Pl}$ is the Planck length. We might postulate that these fluctuations of the spacetime geometry themselves set the averaging scale, in which case $s \approx \ell \approx \ell_{\rm Pl}$. This leads to a picture in which spacetime is stochastic on the Planck scale, along the lines originally proposed by Wheeler. The notion of "spacetime foam" is usually put forward in a context in which gravity is quantized, but it is of interest to note that passive metric fluctuations alone could induce Planck-scale stochasticity.

4. SUMMARY AND DISCUSSION

Let us summarize the basic ideas discussed in this paper. The components of the stress tensor for a quantum field, e.g., the energy density, are formally divergent, but can be rendered finite by a simple subtraction. This subtraction amounts to resetting the zero of energy density to be that of the Minkowski vacuum state, and is the removal of a state-independent quantity. The resulting energy density in the case of the Casimir effect is the physical manifestation of the fluctuations of quantum fields. Although the mean value of the fields themselves are zero, the mean values of the squares of fields can be nonzero.

It is natural to go to the next step and inquire about the fluctuations of the stress tensor itself. If we wish to determine the degree to which the energy density is fluctuating, one wants $\langle \rho^2 \rangle$, the mean squared energy density. Here a fundamental problem arises: This quantity is not only infinite, but cannot be made finite by a state-independent subtraction as was the case for the energy density itself. This is due to the S_1 cross term of Eq. (5), which is both singular and state dependent. Two viewpoints were considered above. One is to drop this term and deal only with the finite, fully normal ordered energy density. This leads to the correct classical limit, Eq. (8), as well as to finite fluctuations in nonclassical states. The other, more radical, viewpoint is that the cross term should be retained. In this view, the notion of squared energy density ceases to have meaning, and one can only talk about space or time integrals which need to be defined by a subtle integration by parts.

There are several unsolved problems which have been touched upon in the preceding discussion. One of these is to understand better the physics of the S_1 cross term. If this term is to be taken seriously, it will require rethinking of the nature of spacetime on small scales. The picture which emerges from this term is that of a stress tensor which is undergoing increasingly violent fluctuations as one goes to smaller and smaller length scales. Nonetheless, the integrated effect of these fluctuations is finite and often rather small. If the spacetime geometry is being driven by these violent fluctuations, then it, too, must have a stochastic character on small scales, yet one that does not usually manifest itself. If the metric were to fluctuate excessively, observable effects such as the scattering of light from distant sources would have been noticed. An outstanding problem is the challenge of quantifying these effects and placing constraints from observation on the degree of metric fluctuations [13, 39].

One area where stress tensor fluctuations may play an important role is in cosmology. Density and gravity wave perturbations may have been created in the early universe, and then played a significant role in its subsequent evolution. Some models of galaxy formation [40] trace the origin of galaxies to quantum fluctuations during a period of inflationary expansion. Several authors [30, 41, 42] have discussed models in which stress tensor fluctuations defined by a version of normal ordering play a role. However, if the cross term is included, the problem becomes much more complicated and has not yet been addressed.

Another unsolved problem is that of better understanding the fluctuating forces on material bodies due to stress tensor fluctuations. It is reasonable to expect that the same principles should apply when the stress tensor is the source of gravity and when it is the agent for producing a force on an object. This raises the exciting possibility of experimentally observing the effects of the cross term. As noted above, these effects take on an added significance for builders of laser interferometer detectors of gravity waves. If one were to detect the effects of the cross term on a real mirror, it would indirectly lead to insights about the quantum nature of the gravitational field and about the small-scale structure of spacetime.

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